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Optimal Design of Composite Structures by Advanced Mixed Integer Nonlinear Optimization

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Optimal Design of Composite Structures by Advanced Mixed Integer Nonlinear Optimization

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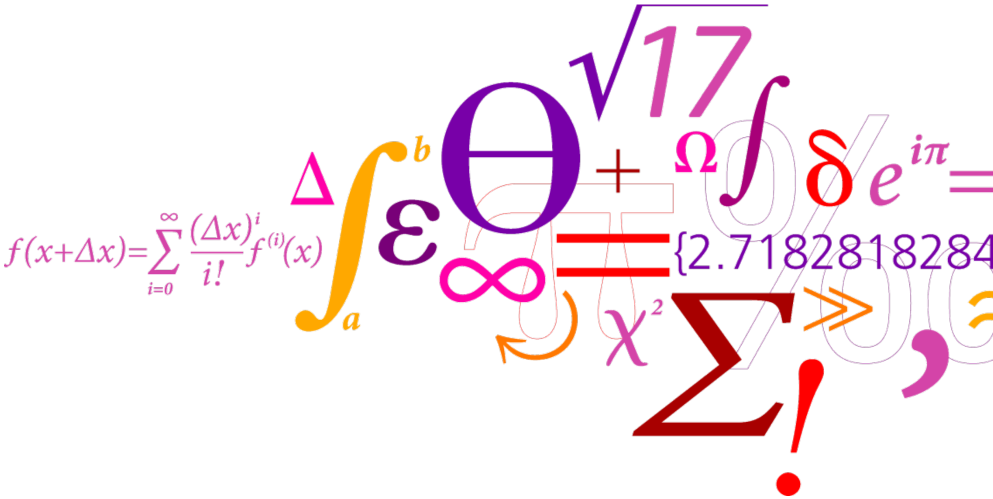

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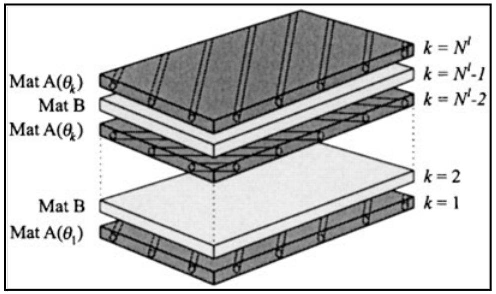
Composite Materials

Composite Material



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Fibres

- Glass Fibres
- Carbon Fibres
- Cellulose Fibres
- and more …

Glass fibres

\+

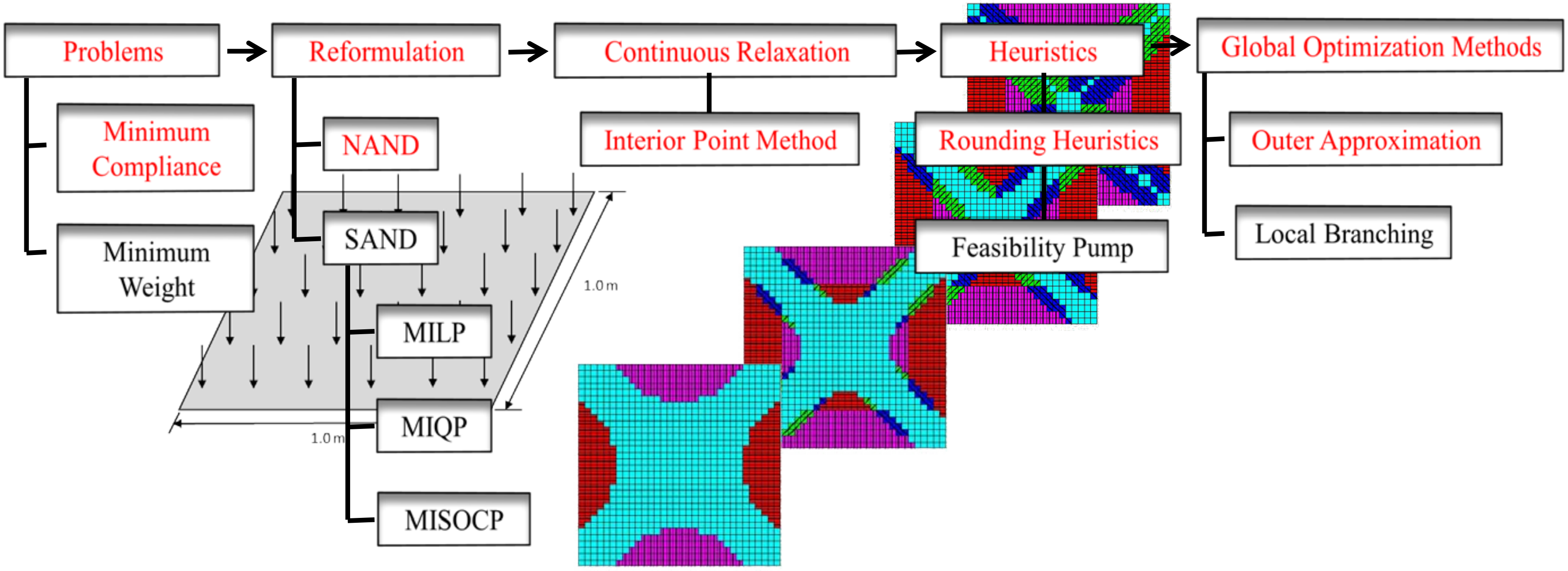
Matrix

- Polymers
- Metals
- Ceramics

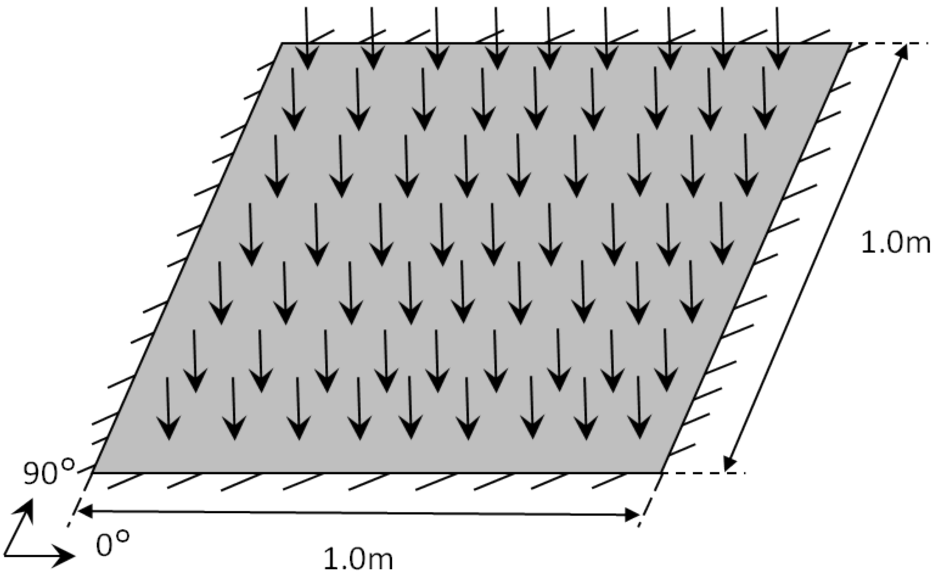
Polyester matrix

Optimal Design of Laminated Composite Structures

- Design Parameterization
  - Number of Layers
  - Fiber Orientation (the considered case)
  - Layer Thickness



Numerical Example – Multi Layered Clamped Plate

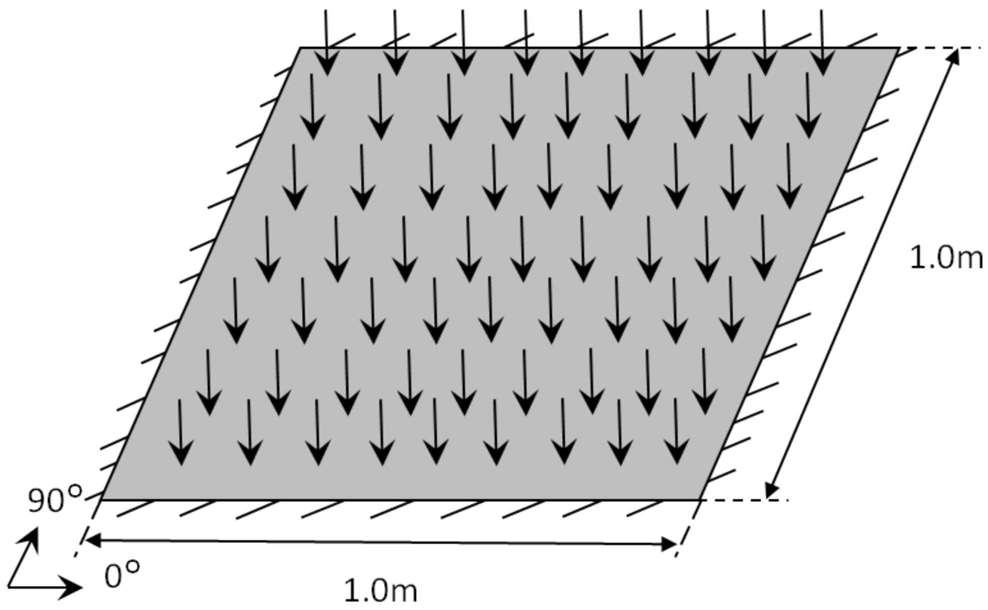


- Distributed Out Of Plane (Static) Loading
- Clamped plate 1.0m x 1.0m x 0.005m
- 8 layers of equal thickness
- 5 candidate materials
  - Orthotropic - Glass fiber reinforced epoxy

    { -45°, 0°, 45°, 90° }
  - Isotropic polymeric foam (35% of the domain)

	Foam	**Glass Epoxy**
E_x [Pa]	$1.25 \cdot 10^8$	$5.4 \cdot 10^{10}$
$E_y = E_z$ [Pa]		$1.8 \cdot 10^{10}$
$G_{xy} = G_{yz} = G_{xz}$ [Pa]		$0.9 \cdot 10^{10}$
Poisson's ratio	0.3	0.25
Density [kg/m³]	100	1900

Numerical Example – Multi Layered Clamped Plate



- All materials behave linearly elastic
- Finite Element Formulations
  - Equivalent Single Layer Theory
  - First Order Shear Deformation Theory
  - Q9 plate/shell elements

Minimum Compliance (Maximum Stiffness)

minimize	Compliance
subject to	Equilibrium equations
	Weight constraint
	Single material selection per layer and element (DMO)

$$
\begin{aligned}
\underset{x,u}{\text{minimize}} \quad & \sum_{l=1}^{L} w_l f_l^T u_l \\
\text{subject to} \quad & \underline{K(x)u_l - f_l = 0,} \qquad l = 1, ..., L \\
& m(x) \leq m_c, \\
& \sum_{i=1}^{n} x_{ijk} = 1, \qquad \forall (j,k) \\
& x_{ijk} \in \{0,1\}, \qquad \forall (i,j,k)
\end{aligned}
$$

- Difficult combinatorial problem.

- The stiffness matrix is
  - linear in the design variables.
  - symmetric and positive definite.

$$K(x) = \sum_{ijk} x_{ijk} K_{ijk} = \sum_{ijk} x_{ijk} B_j^T C_{ik} B_j = \sum_j B_j^T \left(\sum_{ik} x_{ijk} C_{ik}\right) B_j$$

- The nodal displacements are eliminated from the problem formulation.

$$u_l = K(x)^{-1} f_l$$

Reformulation - Nested Analysis and Design

- The original non-convex 0-1 problem is reformulated as a 0-1 program with a convex objective function.

minimize	Compliance
subject to	Weight constraint
	Single material selection per layer and element (DMO)

$$
\begin{aligned}
\underset{x}{\text{minimize}} \quad & c(x) = \underline{\sum_{l=1}^{L} w_l f_l^T K(x)^{-1} f_l} \\
\text{subject to} \quad & m(x) \leq m_c, \\
& \sum_{i=1}^{n} x_{ijk} = 1, \qquad \forall (j,k) \\
& x_{ijk} \in \{0,1\}, \qquad \forall (i,j,k)
\end{aligned}
$$

- The reformulated 0-1 program includes only the binary design variables.

- The equilibrium equations are now solved as part of computing the objective function.

Continuous Relaxation - Interior Point Method

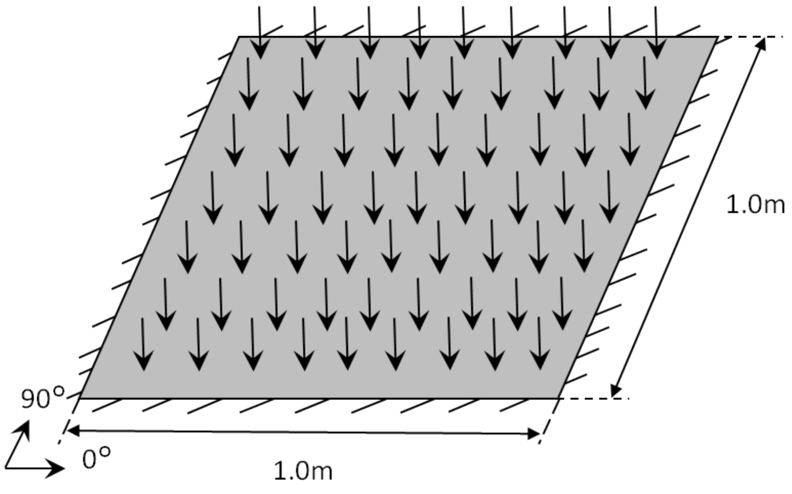
- By relaxing the integer constraints on the design variables we get the continuous relaxation of the minimum compliance problem.

minimize	Compliance
subject to	Weight constraint
	Single material selection per layer and element (DMO)

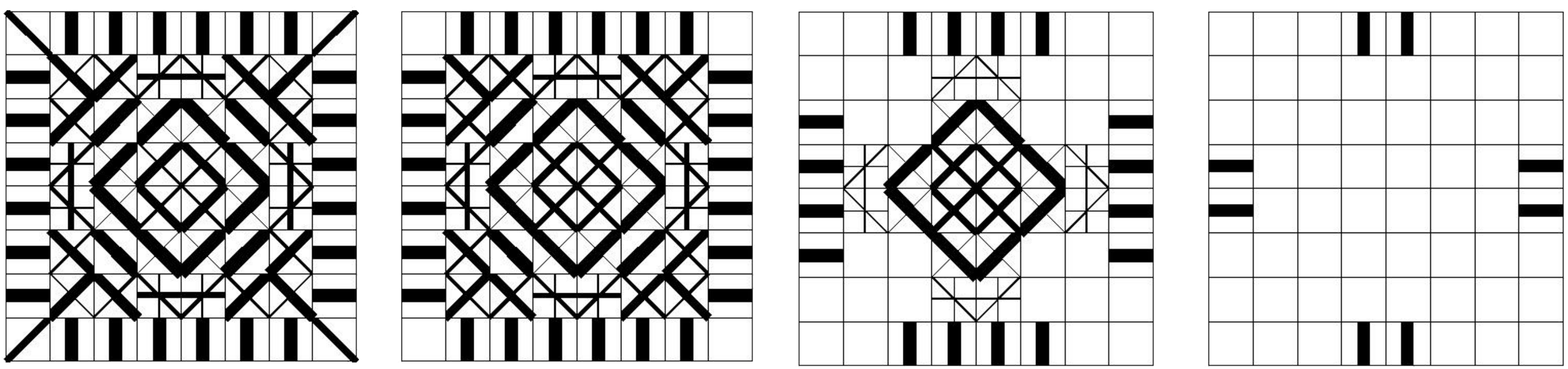
$$
\begin{aligned}
\underset{x}{\text{minimize}} \quad & c(x) = \sum_{l=1}^{L} w_l f_l^T K(x)^{-1} f_l \\
\text{subject to} \quad & m(x) \le m_c \\
& \sum_{i=1}^{n} x_{ijk} = 1, && \forall (j,k) \\
& \underline{x_{ijk} \ge 0,} && \forall (i,j,k)
\end{aligned}
$$

- The continuous problem has a non empty feasible set.
- There is at least one optimal solution of the continuous problem.
- The continuous relaxation of the mixed integer problem is solved by a primal-dual interior point method.

Numerical Example – Multi Layered Clamped Plate
Continuous Relaxation



- Material distribution of the top four layers (due to symmetry) for a mesh discretization of 64 elements.
- The primal-dual interior point method converged in 18 iterations.
- The primal-dual interior point method has the ability to react swiftly to changes of scale of the problem.



Layer 1 *Layer 2* *Layer 3* *Layer 4*

Heuristic

minimize Distance to 0-1 design

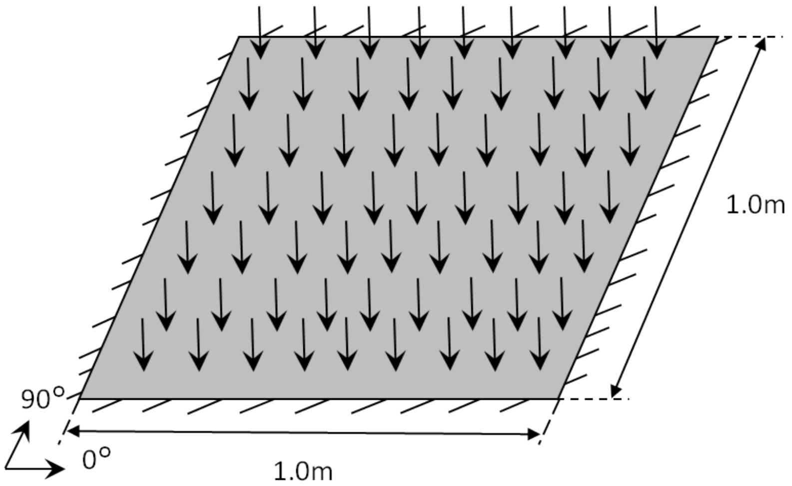
subject to Weight constraint

Single material selection per layer and element (DMO)

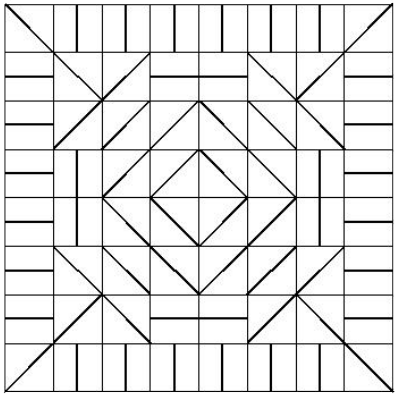
$$
\begin{aligned}
\underset{x}{\text{minimize}} \quad & \|x^* - x\|_1 \\
\text{subject to} \quad & m(x) \leq m_c \\
& \sum_{i=1}^{N^i} x_{ijk} = 1, \qquad \forall (j,k) \\
& x_{ijk} \in \{0,1\}, \qquad \forall (i,j,k)
\end{aligned}
$$

- Rounds the optimal solution of the continuous relaxation .
- The method is guaranteed to find a feasible design.
- There are no guarantees on the quality of the obtained design.

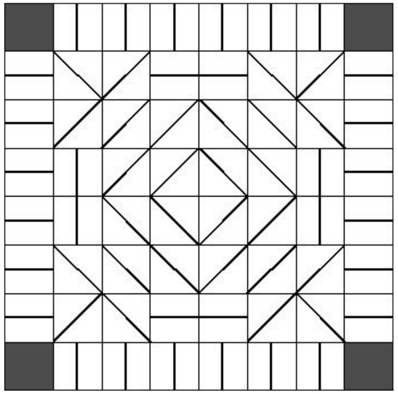
Numerical Example – Multi Layered Clamped Plate
Rounding Heuristic



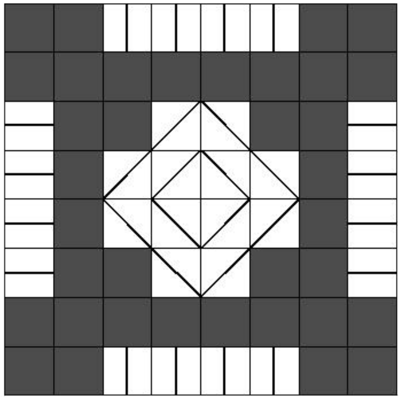
- Material distribution of the top four layers (due to symmetry) for a mesh discretization of 64 elements and a relative optimality gap of 1.23%.
- The optimized designs use the soft material to form a sandwich structure, leading to a high bending stiffness to weight ratio for the composite.
- The fibre angles are oriented according to the principal stress directions.



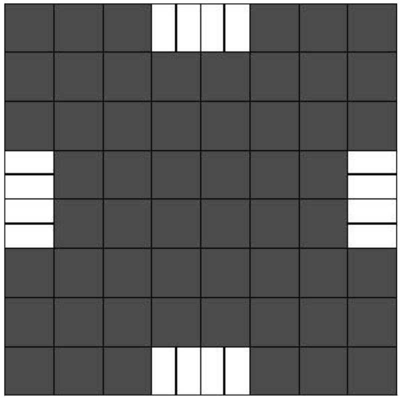
Layer 1



Layer 2



Layer 3



Layer 4

Global Optimization - Outer Approximation

- Approximate the non-linear objective and constraint functions with linear functions.
- Solve a sequence of linear mixed 0-1 problems.
- Guaranteed to converge to global minimizer.

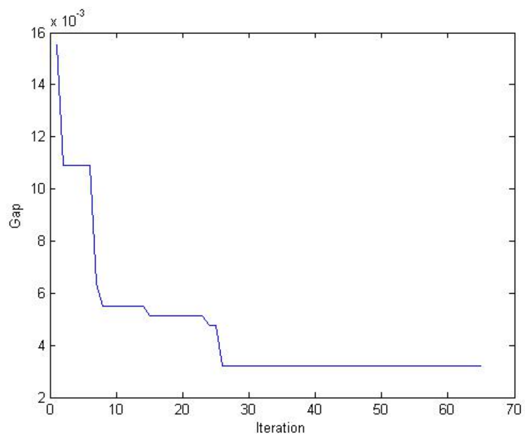
minimize Compliance

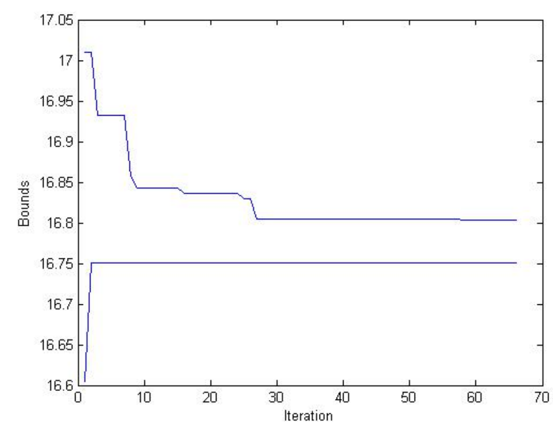
subject to Compliance constraint

Weight constraint

Single material selection per layer and element (DMO)

$$
\begin{aligned}
\underset{\eta,x}{\text{minimize}} \quad & \eta \\
\text{subject to} \quad & \underline{c(x^p) + (\nabla c(x^p))^T (x - x^p) - \eta \le 0, \qquad p = 1, \dots, P} \\
& m(x) \le m_c, \\
& \sum_{i=1}^{N^i} x_{ijk} = 1, \qquad \forall (j,k) \\
& x_{ijk} \in \{0,1\}, \qquad \forall (i,j,k) \\
& \eta \ge 0
\end{aligned}
$$





THANK YOU FOR YOUR ATTENTION